Patterns to Symbols: Algebra

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No Child LEFT BEHIND

Patterns

- **All [life] is pattern … but we can't always see the pattern when we're part of it.**
  - *Belva Plain*
- **Art is the imposing of a pattern on experience, and our aesthetic enjoyment is recognition of the pattern.**
  - *Alfred North Whitehead*
- **When will we be done with patterns and go back to math?**
  - *Unknown 5th grader*

Why focus on patterns?

- Disappointing results on national comparisons point to our students' difficulty with algebra
- "The transition from arithmetic to algebra is often not an easy one" (*Adding It Up*)
- "It is proposed that the artificial separation of arithmetic and algebra deprives students of powerful schemes for thinking about mathematics in the early grades and makes it more difficult for them to learn algebra in the later grades." (Carpenter & Levi, 2000)

Why focus on patterns?

- **To address this, we need to focus on algebra in the elementary years (This does not mean Algebra 1 in 3rd grade.)**
- **Algebraic thinking at the elementary level**
  - Making generalizations
  - Using symbols
- **There are numerous types of generalizations and numerous uses of symbols**

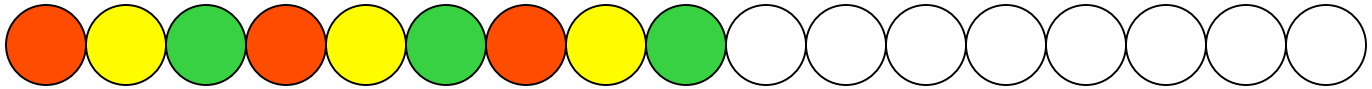
Why focus on patterns?

- One of the easiest ways to begin to think about teaching generalization is through the use of patterns

- In NCTM's *Principles and Standards for School Mathematics*, "Understand patterns, relations, and functions" is a standard at all grade levels

A pattern is a pattern is a pattern

- Kindergarteners are asked to make patterns and to continue a pattern by coloring



- So by 3rd grade we better be asking a little more… Making the steps toward generalizing

NCTM Standards show the progression

- **Grades Pre-K–2**
  - Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another
  - Analyze how both repeating and growing patterns are generated
- **Grades 3–5**
  - Describe, extend, and make generalizations about geometric and numeric patterns
  - Represent and analyze patterns and functions, using words, tables, and graphs
- **Grades 6–8**
  - Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

Let's look at some patterns

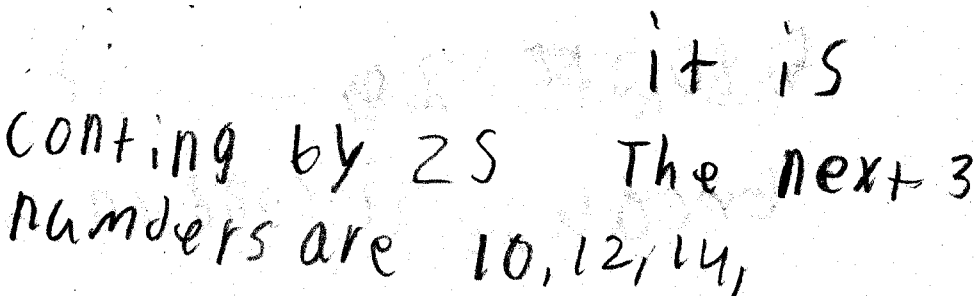
- 2, 4, 6, 8, …
- 5, 10, 15, 20, …
- 5, 8, 11, 14, …
- 1, 2, 4, 7, 11, …

2, 4, 6, 8, …

- What are the next three numbers in the pattern? How do you know?

2, 4, 6, 8, …

- **What are the next three numbers in the pattern? How do you know?**
  - William Grade 3



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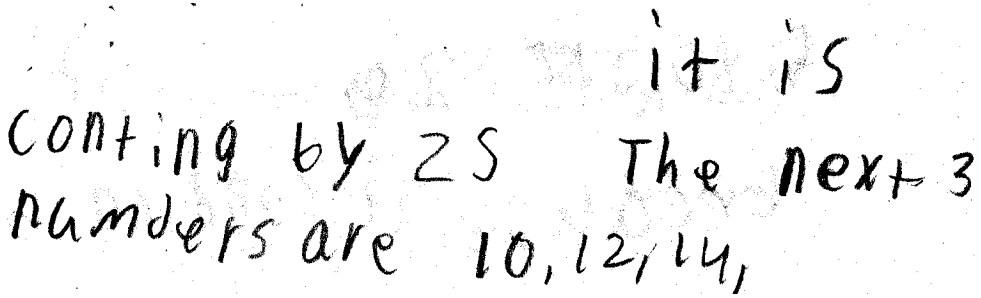
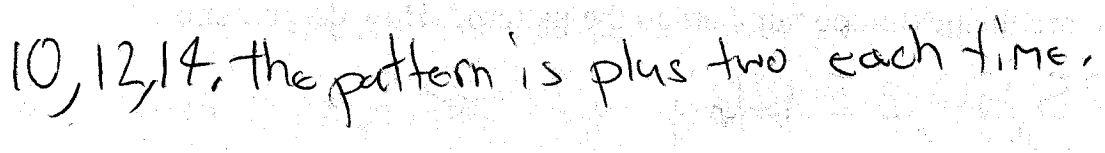


  - Claire Grade 6



2, 4, 6, 8, …

- Would 21 ever be in the pattern? How do you know?

2, 4, 6, 8, …

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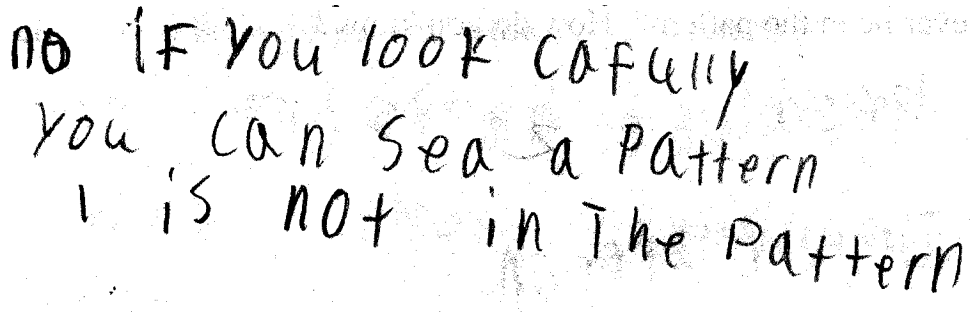

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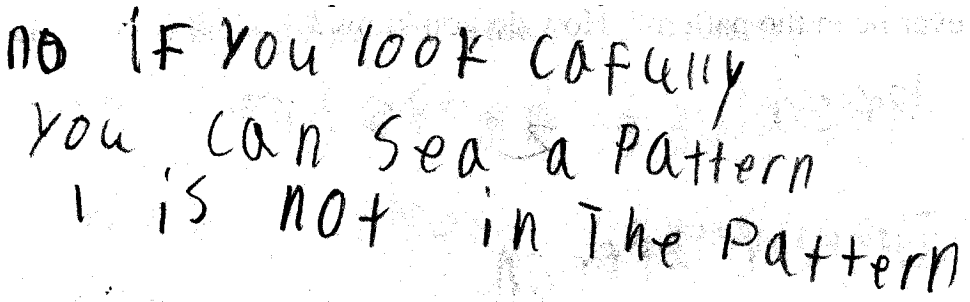

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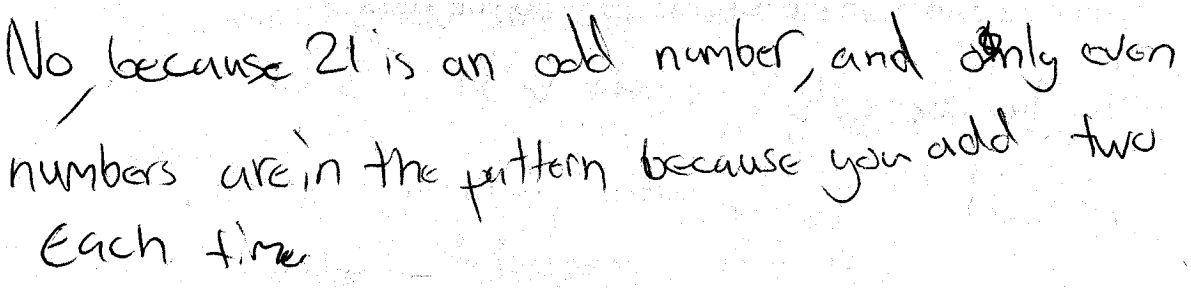

2, 4, 6, 8, …

- **If I give you a random number, how can you tell if that number will be in the pattern?**

2, 4, 6, 8, …

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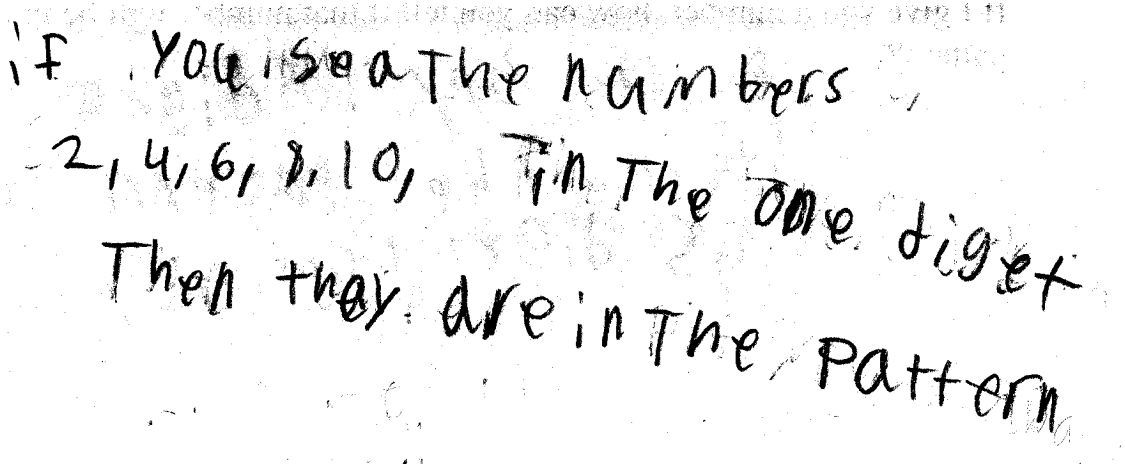

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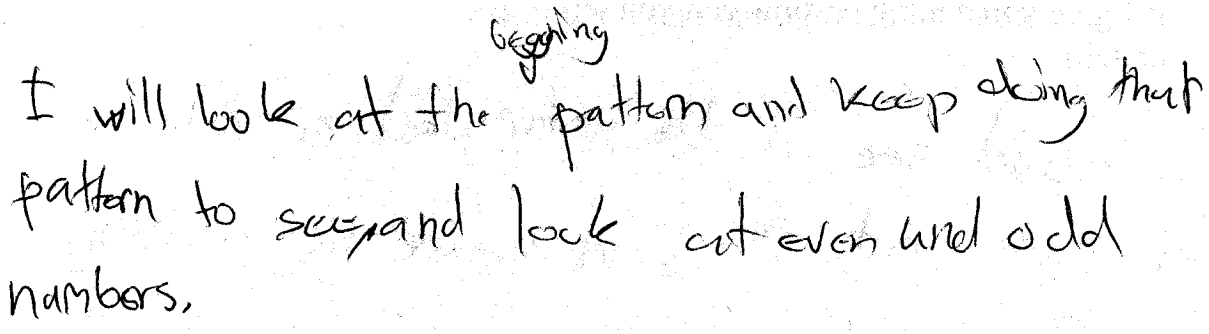
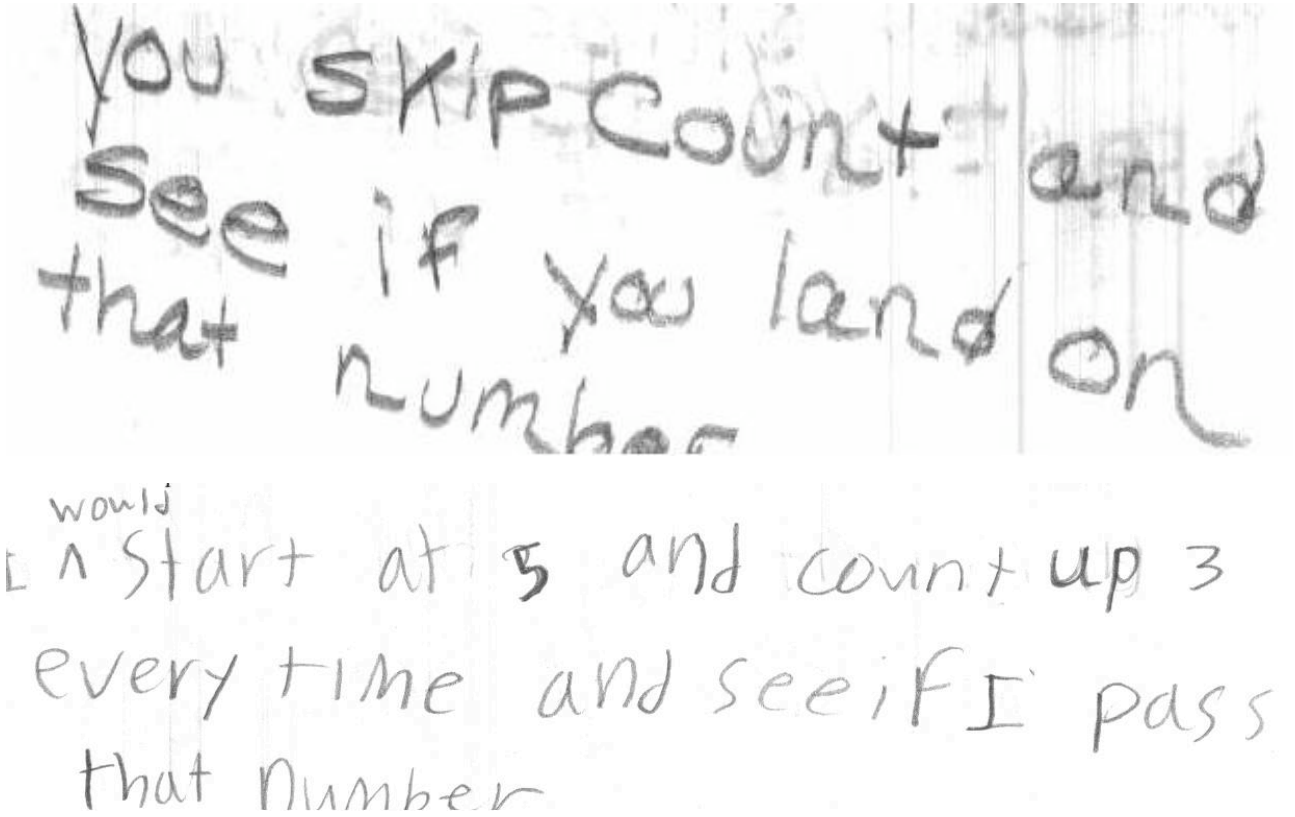


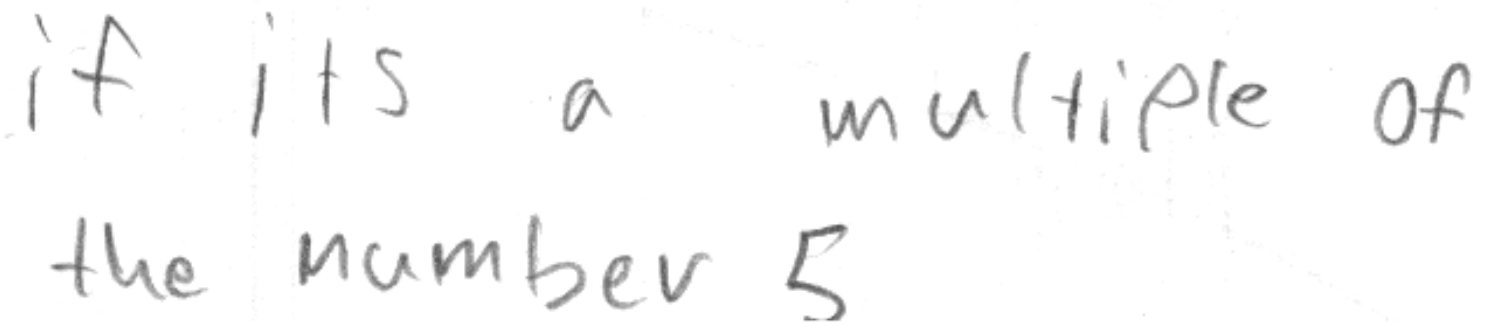
Types of generalization

- **Current – next or recursive**
  - Add two each time



Types of generalization

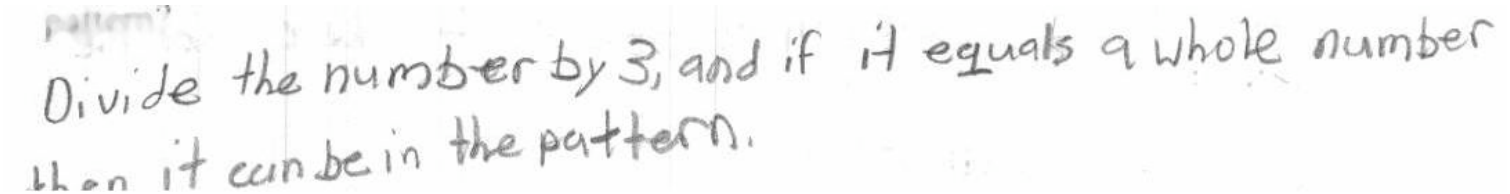
- **Direct**
  - 5 x n
  - 5 + 3n



Errors in generalizing

- **Over-generalizing**

  (for the pattern 5, 8, 11, …)

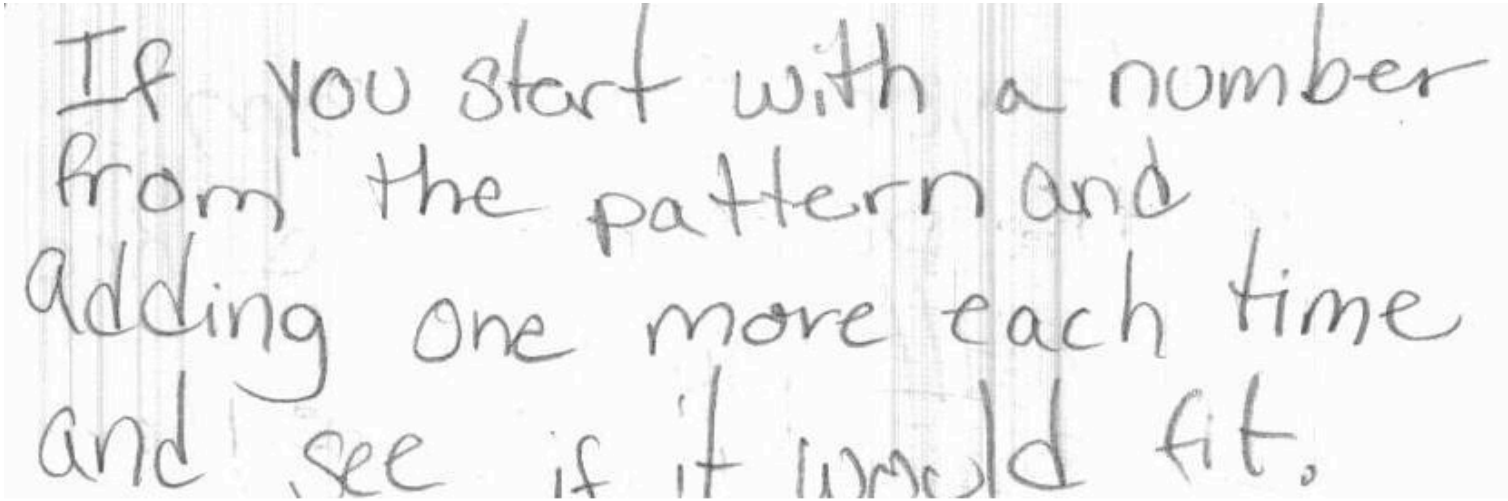


1, 2, 4, 7, 11…

summer workshop

What about harder patterns?

- **Verbal description alone is appropriate at this level**



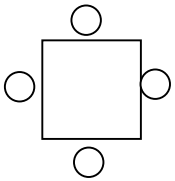
Patterns from a context

- **Table problem**
  - A problem to consider teaching strategies
- **Snake problem**
  - A problem to really think about patterns

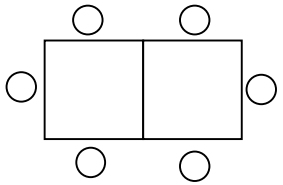
Table Problem

For a special dinner, St. Patrick's is using square tables that can be put together to make rectangles.

One table can seat 4 people



Two tables can seat 6 people



Three tables can seat 8 people

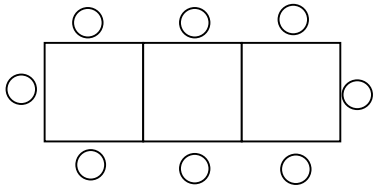


Table problem

- **How many people can be seated at 4 tables?**
  - How could we help a student stuck at this point?

- **How many tables would be needed to seat 16 people? How do you know?**
  - What are we looking for in an explanation?

Table problem

- **What are some possible numbers of people where there would be seats left over? How do you know?**
  - This is the first step in generalizing, noticing what is and isn't in the pattern.
  - What description might a 3rd grader give? A 5th grader?

Table problem

- **Mr. Barrett says that he will tell you how many tables he needs, and then he will ask you how many people are coming. How would you figure out how many people are coming?**
  - Possible Strategies?

Table problem

- **Current: next or recursive**
  - Each table adds 2 more people
  - The number of people is always even
- **Direct**
  - 2 + number of tables x 2

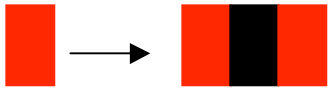
tables	people
1	4
2	6
3	8
4	10

Snakes

- Instead of being given a pattern, we are given a starting point and a rule

Starting point:

Rules:



Snakes

Pattern	Number of Red Rings	Number of Black Rings	Number of Rings
	1	0	1
	2	1	3
	4	3	7
	8	7	15
	16	15	31

Pattern problems

- **The tables and snakes problems are different from the number pattern problems**
  - A series of instances rather than a sequence
  - Can be modeled
  - Generalization is within the context of the problem

What does generalizing mean at grades 3–6

- **Explaining patterns in sequences of numbers**
  - Relating to multiples
  - Noting constant or variable change (slope)
  - Paying attention to starting points (intercept)
  - Identifying which numbers are and aren't in the sequence and why

What does generalizing mean at grades 3–6

- **Explaining patterns in context**
  - Relate how numbers change to where they come from
  - Move fluidly from pattern as numbers to what the pattern means for the context
  - Analyze numbers from context as a sequence
  - Move pattern beyond context

What to do?

- **Talk about sequences, more than just what comes next**
  - Relate it to work with multiplication
  - Work on a hundreds chart
  - Given a rule and a starting number, create the sequence
- **Work with pattern problems**
  - What's my rule?
  - Explore both constant and variable relationships

What to do in the fall?

- Distributing activities via the Web
- Providing support via the Web

References

- National Research Council. (2001). *Adding It Up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
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- Roodhardt, A., Kindt, M., Burrill, G., & Spence, M. (1997). *Patterns and Symbols.* In National Center for Research in Mathematical Sciences Education and Freudenthal Institute (Eds.), Mathematics in Context: A connected curriculum for Grades 5-8. Chicago, IL: Encyclopedia Britannica Educational Corporation. ISBN 0030712831